Gerrymandering in Utah

Monte-Carlo Markov Chain Methods and Metropolis-Hastings Sampling

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Outline

Markov Chain Monte Carlo

- Utah has about 3000 voting precincts, and 4 congressional districts
- Idea: create a Markov chain with the desired stationary distribution
- Advantages: we do not have to explore every possible state
- Disadvantages: may take long to converge

Metropolis-Hastings Construction

- Let S be the set of all legal districting plans, and let $p_X(s) = P(X = s)$ be the probability mass function describing this distribution.
- Construct a Markov chain with equal transition probabilities, but where proposals are accepted with probability

$$A(s', s) = \min\left(\frac{p_X(s')}{p_X(s)}, 1\right)$$

Utah Proposition 4

"In drawing the districts, the commission and the Legislature must abide by certain standards, to the extent practicable, including:

- Preserving equal populations among districts.
- Keeping municipalities and counties together.
- Creating districts that are compact and contiguous.
- Respecting traditional neighborhoods and communities of interest.
- Following geographic features and natural barriers.

The law prohibits drawing districts to unduly favor or disfavor any incumbent, candidate, or political party.

Score Functions

To quantify gerrymandering in North Carolina, Dr. Jonathan Mattingly created score functions for various metrics

- Compactness

$$J_c(\xi) = \sum_i \frac{\text{perimeter}(D_i(\xi))^2}{\text{area}(D_i(\xi))}$$

- Population

$$J_p(\xi) = \sqrt{\sum_i \left(\frac{\text{pop}(D_i(\xi))}{\text{pop}_{\text{ideal}}} - 1 \right)^2}$$

- Split counties

$$J_s(\xi) = \text{ doubly-split counties} \cdot W_2(\xi) + M \text{ triply-split counties} \cdot W_3(\xi)$$

where

Bibliography

https://betterboundaries.org/faq/
Mattingly, Jonathan C. "Quantifying Gerrymandering in North Carolina. January 9, 2018.